METHOD OF GENERALIZED SIMILITUDE IN PROBLEMS OF THE MOVEMENT OF AN IMPERFECT GAS IN A THIN LAYER

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The article presents results of the application, to problems of gas lubrication, of the method of generalized similitude for solving problems of boundary-layer theory.

1. In the theory of lubrication, it is customary to assume that inertial forces are negligibly small compared to viscous friction forces. On the basis of this, equations may not include terms representing acceleration of friction, i.e., it may be assumed that the pressure gradient necessary to create buoyancy is developed by viscous friction alone. This assumption is based on the triviality of the so-called lubrication Reynolds number Re*, equal to the product of the regular Reynolds number ($Re = U\delta_0/\nu$) and the relative thickness of the lubricant layer (δ_0/L). As a result, $Re^* = Re\delta_0/L \ll 1$, which justifies ignoring inertial forces. The equations are simplified considerably in this case. They are integrated and their solution is represented in the form of a single pressure equation, which is nonlinear in the case of gas lubrication. Methods of integrating the latter have been well developed and are contained, e.g., in [1, 2].

However, the number Re* may be of the order of unity or even greater ($Re^* \geqslant 1$) for sliding bases on high-speed machines and supports with a large relative thickness of the lubricant layer. Inertial forces cannot be ignored in these cases, but rather must be taken into account in final design of the support, i.e., the convective (nonlinear) part of the equations must be retained.

In this case, the flow of viscous liquids and gases is described by parabolic equations [2], i.e., the same type of equations as describe the boundary layer. The difference between these groups of equations lies in the fact that problems of boundary-layer theory generally involve calculation of nominal thicknesses of the layer from an assigned distribution of pressure or velocity on the external boundary of the layer, while lubrication theory generally entails the solution of the inverse problem — calculation of the pressure distribution for a known thickness of lubricant layer.

The survey article [3] examined methods of designing sliding supports with allowance for lubricant inertia. In addition to this, we should note the possibility of using integral relations [4].

The present article, using two-dimensional problems of the steady movement of a gaseous lubricant with allowance for inertial forces, proposes the use of the method of generalized (parametric) similitude advanced by Loitsyanskii to solve problems of boundary-layer theory [5]. We explained the essence of the method in [6] as it pertains to problems of the hydrodynamic theory of lubrication: equations are formulated for current and enthalpy functions in variables which represent a set of parameters containing the geometric, kinematic, and thermodynamic conditions of lubrication problems, so that the final form of these equations does not contain specific features of the problem in explicit form. Since the formulated equations and their solutions turn out to be general for all problems, the equations are called universal equations. The solutions to these equations can be obtained, tabulated, and shown in a form convenient for ready practical reference. In specific calculations, a solution will be found to problems of lubrication theory which consists of integrating a pressure equation that generalizes the Reynolds equation for the case of allowing for inertial forces and transforms into the latter as $Re^* \to 0$.

2. General Formulation of the Problem. Assuming the triviality of the relative

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thickness of the lubricant layer and smoothness of its contour, the steady movement of the lubricant is described by the system of equations

$$\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}=0;\quad u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y};$$
$$\frac{\partial p}{\partial y}=0;\quad u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}=\frac{u}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\Pr}\frac{1}{\rho}\frac{\partial}{\partial y}\mu\frac{\partial h}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial u}{\partial y}\right)^2 \tag{1}$$

with the boundary conditions

$$u=U,\ v=0,\ h=h_1 \text{ for } y=0;\ u=v=0,\ h=h_2$$
$$\text{for } y=\delta(x);\ \text{for } x=x_0\ u=u_0(y),\ h=h_0(y). \tag{2}$$

To close the system of equations, we use the generally accepted assumptions in [5]:

the state of the gas conforms to Clapeyron's law

$$\frac{p}{\rho}=\frac{k-1}{k}h; \tag{3}$$

the dynamic viscosity coefficient is a power function of the absolute temperature (enthalpy):

$$\frac{\mu}{\mu_0}=\left(\frac{h}{h_0}\right)^n=H^n; \tag{4}$$

the Prandtl number $\Pr = \mu c_p/\lambda$ is independent of the temperature and is a physical constant of the gas.

As in the case of boundary-layer equations, Eqs. (1) may be put in a form close to the form of the equations of motion of an incompressible liquid. To this end, we will use Dorodnitsyn's variables [5]

$$\xi=\int_{x_0}^{x}\frac{p}{p_0}dx;\quad \eta=\int_0^y\frac{\rho}{\rho_0}dy. \tag{5}$$

In these variables, Eqs. (1) and the boundary conditions take the form

$$\frac{\partial u}{\partial \xi}+\frac{\partial \tilde{v}}{\partial \eta}=0;\quad u\frac{\partial u}{\partial \xi}+\tilde{v}\frac{\partial u}{\partial \eta}=-\frac{1}{\rho}\frac{dp}{d\xi}+\nu_0\frac{\partial}{\partial \eta}H^{n-1}\frac{\partial u}{\partial \eta};$$
$$u\frac{\partial h}{\partial \xi}+\tilde{v}\frac{\partial h}{\partial \eta}=\frac{u}{\rho}\frac{dp}{d\xi}+\frac{\nu_0}{\Pr}\frac{\partial}{\partial \eta}H^{n-1}\frac{\partial h}{\partial \eta}+\nu_0 H^{n-1}\left(\frac{\partial u}{\partial \eta}\right)^2; \tag{6}$$

$$\eta=0,\ u=U,\ \tilde{v}=0,\ h=h_1;\ \eta=\eta_\delta,\ u=\tilde{v}=0,$$
$$h=h_2;\ \text{for } \xi=\xi_0\ u=u_0(\eta),\ h=h_0(\eta). \tag{7}$$

Here

$$\tilde{v}=vH^{-1}+u\frac{p_0}{p}\frac{\partial \eta}{\partial x};\quad \eta_\delta=\int_0^\delta\frac{\rho}{\rho_0}dy. \tag{8}$$

Using the formulas

$$u=\frac{\partial \psi}{\partial \eta};\quad \tilde{v}=-\frac{\partial \psi}{\partial \xi} \tag{9}$$

we make the transformation in Eqs. (6) and boundary conditions (7) to the current function ψ:

$$\frac{\partial \psi}{\partial \eta}\frac{\partial^2 \psi}{\partial \xi\partial \eta}-\frac{\partial \psi}{\partial \xi}\frac{\partial^2 \psi}{\partial \eta^2}=-\frac{1}{\rho}\frac{dp}{d\xi}+\nu_0\frac{\partial}{\partial \eta}H^{n-1}\frac{\partial^2 \psi}{\partial \eta^2};$$
$$\frac{\partial \psi}{\partial \eta}\frac{\partial h}{\partial \xi}-\frac{\partial \psi}{\partial \xi}\frac{\partial h}{\partial \eta}=\frac{1}{\rho}\frac{\partial \psi}{\partial \eta}\frac{dp}{d\xi}+\frac{\nu_0}{\Pr}\frac{\partial}{\partial \eta}H^{n-1}\frac{\partial h}{\partial \eta}+\nu_0 H^{n-1}\left(\frac{\partial^2 \psi}{\partial \eta^2}\right)^2; \tag{10}$$

$$\psi=0,\ \frac{\partial\psi}{\partial\eta}=U,\ h=h_1 \text{ for } \eta=0;\ \psi=Q,\ \frac{\partial\psi}{\partial\eta}=0,\ h=h_2 \text{ for } \eta=\eta_\delta;$$

$$\psi=\psi_0(\eta),\ h=h_0(\eta) \text{ for } \xi=\xi_0, \tag{11}$$

where

$$Q=\int_0^{\eta_\delta} u\,d\eta=\frac{Q_0}{\rho_0}=\text{const};$$ Q_0 is the mass flow rate of the lubricant.

3. Universal Equations. We will show that the above system of equations (10) and boundary conditions (11) can actually be written in the aforementioned universal form. To this end, let us make the transformation to the new variables, Φ, and $\tilde{H}$:

$$\varphi=\frac{\eta}{\eta_\delta(\xi)};\ \psi(\xi,\ \eta)=Q\Phi(\xi,\ \varphi);\ h(\xi,\ \eta)=h_1+(h_2-h_1)\tilde{H}(\xi,\ \varphi). \tag{12}$$

Using the usual formulas for such a transformation

$$\frac{\partial}{\partial\xi}=\frac{\partial}{\partial\xi}-\varphi\frac{1}{\eta_\delta}\frac{d\eta_\delta}{d\xi}\frac{\partial}{\partial\varphi},\quad \frac{\partial}{\partial\eta}=\frac{1}{\eta_\delta}\frac{\partial}{\partial\varphi},\quad \frac{\partial^2}{\partial\eta^2}=\frac{1}{\eta_\delta^2}\frac{\partial^2}{\partial\varphi^2}, \tag{13}$$

we have the system of equations

$$\frac{\partial}{\partial\varphi}H^{n-1}\frac{\partial^2\Phi}{\partial\varphi^2}+\frac{Q}{\nu_0}\frac{d\eta_\delta}{d\xi}\left(\frac{\partial\Phi}{\partial\varphi}\right)^2+\frac{Q\eta_\delta}{\nu_0}\left[\frac{\partial^2\Phi}{\partial\varphi^2}\frac{\partial\Phi}{\partial\xi}-\frac{\partial\Phi}{\partial\varphi}\frac{\partial^2\Phi}{\partial\xi\partial\varphi}\right]=HP;$$

$$\frac{1}{\Pr}\frac{\partial}{\partial\varphi}H^{n-1}\frac{\partial\tilde{H}}{\partial\varphi}+\frac{Q^2}{\Delta h\eta_\delta^2}\left[HP+H^{n-1}\left(\frac{\partial^2\Phi}{\partial\varphi^2}\right)^2\right]+\frac{Q\eta_\delta}{\nu_0}\left[\frac{\partial\tilde{H}}{\partial\varphi}\frac{\partial\Phi}{\partial\xi}-\frac{\partial\Phi}{\partial\varphi}\frac{\partial\tilde{H}}{\partial\xi}\right]=0 \tag{14}$$

with the boundary conditions

$$\Phi=0,\quad \frac{\partial\Phi}{\partial\varphi}=\frac{U\eta_\delta}{Q},\ \tilde{H}=0 \text{ at } \varphi=0;$$

$$\Phi=1,\quad \frac{\partial\Phi}{\partial\varphi}=0,\ \tilde{H}=1 \text{ at } \varphi=1. \tag{15}$$

In Eqs. (14), we adopt the notation

$$P=\frac{p_0\eta_\delta^3}{\mu_0 Q}\frac{d\ln p}{d\xi}. \tag{16}$$

The equations include dimensionless complexes which themselves include quantities characterizing the kinematic, geometric, and thermodynamic conditions of the flow of the gaseous lubricant. Let us use the complex $f_1=(Q/\nu_0)(d\eta_\delta/d\xi)$ as the basis for constructing the series of complexes

$$f_k=\frac{Q^k\eta_\delta^{k-1}}{\nu_0^k}\frac{d^k\eta_\delta}{d\xi^k}\ (k=0,\ 1,\ 2,\ \ldots), \tag{17}$$

which, by analogy with boundary-layer theory, we will call form-parameters or, simply, parameters. The parameter f_0, equal to unity, is included in the analysis for the sake of the generality of the notation. The derivatives with respect to ξ of this series of parameters give the recurrent relation

$$\frac{Q\eta_\delta}{\nu_0}\frac{df_k}{d\xi}=(k-1)f_1f_k+f_{k+1}=\theta_{k+1}, \tag{18}$$

which relates the derivative of the parameter with the number k to the parameter with the number which is larger by unity, and which lies at the basis of the procedures used to obtain series of parameters. The complexes

$$f_V=\frac{U\eta_\delta}{Q},\quad f_d=\frac{Q^2}{\Delta h\eta_\delta^2} \tag{19}$$

do not form parameter series, since their derivatives are expressed through the complexes themselves and the parameter f_1:

$$\frac{Q\eta_\delta}{\nu_0}\frac{df_V}{d\xi}=f_1f_V,\quad \frac{Q\eta_\delta}{\nu_0}\frac{df_d}{d\xi}=-2f_1f_d \tag{20}$$

and have an independent value. Here, the parameter f_d is an analog of the Eckert number familiar in heat-transfer theory and accounts for the flow of heat to the lubricant layer due to dissipation and compression of the gas.

Assuming the randomness of the continuous functions — differentiable as many times as we please — entering into the determination of the parameters of (17) and (19), we may take them as new independent variables and complete in the equations the final transformation from the parameter space $\{\xi, \varphi\}$ to the parameter space $\{f_k, f_V, f_d, \varphi\}$ using the formulas

$$\frac{Q\eta_\delta}{\nu_0}\frac{\partial}{\partial\xi} = \theta_{k+1}\frac{\partial}{\partial f_k} + f_1 f_V\frac{\partial}{\partial f_V} - 2f_1 f_d\frac{\partial}{\partial f_d} = L, \tag{21}$$

where summation is carried out according to the repeating index k (k = 1, 2, ...). As a result, Eqs. (14) take the universal form

$$\frac{\partial}{\partial\varphi}H^{n-1}\frac{\partial^2\Phi}{\partial\varphi^2} + f_1\left(\frac{\partial\Phi}{\partial\varphi}\right)^2 + \left[\frac{\partial^2\Phi}{\partial\varphi^2}L(\Phi) - \frac{\partial\Phi}{\partial\varphi}L\left(\frac{\partial\Phi}{\partial\varphi}\right)\right] = HP;$$

$$\frac{1}{\Pr}\frac{\partial}{\partial\varphi}H^{n-1}\frac{\partial\tilde{H}}{\partial\varphi} + f_d\left[PH + H^{n-1}\left(\frac{\partial^2\Phi}{\partial\varphi^2}\right)^2\right] + \left[\frac{\partial\tilde{H}}{\partial\varphi}L(\Phi) - \frac{\partial\Phi}{\partial\varphi}L(\tilde{H})\right] = 0 \tag{22}$$

with the boundary conditions

$$\Phi = 0,\quad \frac{\partial\Phi}{\partial\varphi} = f_V,\quad \tilde{H} = 0 \text{ at } \varphi = 0;$$

$$\Phi = 1,\quad \frac{\partial\Phi}{\partial\varphi} = 0,\quad \tilde{H} = 1 \text{ at } \varphi = 1. \tag{23}$$

By virtue of the fact that the pressure is independent of the transverse coordinate, the function P is a function only of the parameters and is determined from the condition $\Phi(1) = 1$.

The point of the parameter space $f_1 = f_2 = \ldots = f_k = \ldots = f_V = f_d = 0$ is a regular singular point of the system (22). System (22) degenerates into the following system at this singular point:

$$\frac{\partial}{\partial\varphi}H_0^{n-1}\frac{\partial^2\Phi_0}{\partial\varphi^2} = P_0 H_0;\quad \frac{\partial}{\partial\varphi}H^{n-1}\frac{\partial\tilde{H}_0}{\partial\varphi} = 0, \tag{24}$$

the solution of which, with conditions (23), allows us to make an approximate substitution for the last condition in (11). Here, we will assume that $f_k = f_V = f_d = 0$ (k = 1, 2, ...), $\tilde{H} = \tilde{H}_0$, $\Phi = \Phi_0$, $P = P_0$.

Given the usual assumption of the theory of gas lubrication on the isothermal nature of flow in the lubricant layer, the need for heat balance in the equation is done away with. Thus, the equation of momentum takes the form (the dots signify derivatives with respect to φ)

$$\dddot{\Phi} + f_1\dot{\Phi}^2 + [L(\Phi)\ddot{\Phi} - L(\dot{\Phi})\dot{\Phi}] = P, \tag{25}$$

which in form is the same as the analogous equation of gas-dynamic lubrication theory [6].

4. Parametric Approximations. In the presence of an infinite series of parameters (17), the differential operators L of (21) represent an infinite sum of derivatives with respect to the parameters. Thus, integration of the universal equations is possible in a practical sense only if we consider a finite number of these parameters in so-called parametric approximations.

Here, the following equation is a single-parameter approximation ($f_V \neq 0$, $f_1 = f_2 = \ldots = f_k = \ldots = 0$):

$$\dddot{\Phi}^{(1)} = P^{(1)}, \tag{26}$$

the solution of which, with allowance for the conditions (23), yields the following expressions for the functions $\Phi^{(1)}$ and $P^{(1)}$:

$$\Phi^{(1)}(\varphi, f_V) = (3 - 2\varphi)\varphi^2 + f_V(1-\varphi)^2\varphi;\quad P^{(1)} = -12\left(1 - \frac{1}{2}f_V\right), \tag{27}$$

corresponding to the inertialess flow of gaseous lubricant normally investigated ($Re^* \ll 1$).

We will designate as a two-parameter approximation of the equation its "interval"

$$\dddot{\Phi}^{(2)}+f_1[\ddot{\Phi}^{(2)}]^2+f_1f_V\left[\ddot{\Phi}^{(2)}\frac{\partial\dot{\Phi}^{(2)}}{\partial f_V}-\dot{\Phi}^{(2)}\frac{\partial\ddot{\Phi}^{(2)}}{\partial f_V}\right]=P^{(2)} \tag{28}$$

with the conditions

$$\varphi=0\quad \Phi^{(2)}=0,\quad \dot{\Phi}^{(2)}=f_V;\quad \varphi=1\quad \Phi^{(2)}=1,\quad \dot{\Phi}^{(2)}=0. \tag{29}$$

With $f_V = 0$, function $\Phi^{(2)}$ should satisfy the equation

$$\dddot{\Phi}_0^{(2)}+f_1[\ddot{\Phi}_0^{(2)}]^2=P_0^{(2)}(f_1) \tag{30}$$

with conditions (29). This approximation will be accurate if function $n_\delta(\xi)$ is linear in the Dorodnitsyn variables. Also, the equation of the n-parameter approximation will generally be exact if this function is a polynomial of the (n − 1) degree in these variables.

The equations in the above approximations with the corresponding conditions give generalized-similar solutions of isothermal problems of the gasdynamic theory of lubrication. The simplest of such generalizations of the concept of similitude is the local-similar solution, presented in [6], of the equation

$$\dddot{\Phi}+f_1\ddot{\Phi}^2=P \tag{31}$$

with conditions (29). In this solution, the partial derivatives with respect to the parameters are neglected. It is easy to see that this approximation is intermediate between single- and two-parameter approximations. In this case, we will call the "interval" of Eq. (25) which includes no derivatives with respect to the parameter f_n (summation in (21) according to k up to n − 1) the local-n-parametric approximation of this equation.

Numerical methods are presently used to solve the equation of the two-parameter approximation (28). We will be limited by the fact that we will represent the solution in the neighborhood of a regular singular point in the form of an expansion into a series in powers of the parameters:

$$\Phi=\Phi_0+\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\cdots\sum_{k_n=1}^{\infty}\cdots\Phi_{k_1,k_2,\ldots,k_n,\ldots}\, f_V^{k_1}f_1^{k_2}f_2^{k_3}\cdots f_{n-1}^{k_n}\cdots,$$
$$P=P_0+\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\cdots\sum_{k_n=1}^{\infty}\cdots P_{k_1,k_2,\ldots,k_n,\ldots}\, f_V^{k_1}f_1^{k_2}f_2^{k_3}\cdots f_{n-1}^{k_n}\cdots, \tag{32}$$

having limited them to intervals of the form (two-parameter approximation):

$$\Phi=\Phi_{00}+\Phi_{01}f_1+(\Phi_{10}+\Phi_{11}f_1)f_V+(\Phi_{20}+\Phi_{21}f_1)f_V^2;$$
$$P=P_{00}+P_{01}f_1+(P_{10}+P_{11}f_1)f_V+(P_{20}+P_{21}f_1)f_V^2. \tag{33}$$

Let us substitute these expansions into Eq. (28) and equate the coefficients at identical combinations of the parameters. Using direct integration to solve the resulting system of ordinary differential equations with conditions which follow from (29) and then substituting the expansions (32) in (29), we obtain the following expressions for the coefficients Φ_{ij} and P_{ij}:

$$\Phi_{00}=(3-2\varphi)\varphi^2;$$
$$\Phi_{01}=-\frac{6}{35}\varphi^7+\frac{3}{5}\varphi^6-\frac{3}{5}\varphi^5+\frac{9}{35}\varphi^3-\frac{3}{35}\varphi^2;$$
$$\Phi_{10}=\varphi(1-\varphi)^2;\quad \Phi_{20}=0;$$
$$\Phi_{11}=\frac{1}{7}\varphi^7-\frac{3}{5}\varphi^6+\frac{9}{10}\varphi^5-\frac{1}{2}\varphi^4-\frac{1}{70}\varphi^3+\frac{1}{14}\varphi^2;$$
$$\Phi_{21}=-\frac{1}{35}\varphi^7+\frac{2}{15}\varphi^6-\frac{7}{30}\varphi^5+\frac{1}{6}\varphi^4-\frac{1}{42}\varphi^3-\frac{1}{70}\varphi^2; \tag{34}$$
$$P_{00}=-12;\quad P_{01}=\frac{54}{35};\quad P_{10}=6;\quad P_{11}=-\frac{9}{35};$$

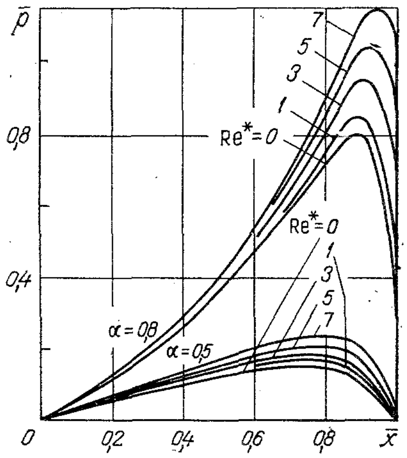

Fig. 1. Distribution of pressure $\bar{p} = (p - p_0)/p_0$ along the lubricant wedge.

$$P_{20} = 0; \quad P_{21} = -\frac{1}{7}. \tag{35}$$

5. Equation for Pressure. Equation (16) is solved in specific calculations of lubricant flow. Allowing for the resulting form of the function in (33) and (35), we must first change over from the Dorodnitsyn variables in this equation to the physical variables x and y. As a result of this transformation, we have the following expressions for the function P and parameters f_V and f_1:

$$P = \frac{12}{q} p\delta^3 \frac{dp}{dx}; \quad f_V = 2\frac{\Lambda}{q} p\delta; \quad f_1 = \frac{1}{2}\frac{q}{\Lambda}\mathrm{Re}^* \frac{(p\delta)'}{p}, \tag{36}$$

where

$$q = \frac{12\mu_0 Q_0 L}{p_0 \rho_0 \delta_0^3}; \quad \Lambda = \frac{6\mu_0 UL}{p_0 \delta_0^2}. \tag{37}$$

Now substituting Eqs. (36) and (37) in (33), we obtain the final equation for pressure. This equation generalizes the Reynolds equation for the case of inertial flow of a gas lubricant:

$$\frac{d}{dx}\left[\frac{p\delta^3}{F_1}\frac{dp}{dx}\right] = 2a_{10}\Lambda \frac{d}{dx}\left[p\delta \frac{F_2}{F_1}\right], \tag{38}$$

where

$$F_1 = a_{00} + a_{11}\mathrm{Re}^*\delta(p\delta)'\left(1 + \frac{1}{2}\frac{a_{01}}{a_{11}}\frac{q}{\Lambda}\frac{1}{p\delta}\right);$$

$$F_2 = 1 + \frac{a_{21}}{a_{10}}\mathrm{Re}^*\delta(p\delta)'. \tag{39}$$

The coefficients a_{ij} are expressed through P_{ij} ($a_{ij} = P_{ij}/12$) and can be regarded not only as coefficients of a series expansion, but also as coefficients of a polynomial of the type (33) – approximating the function P in the solution of a universal equation in a given interval of change in the parameters f_V and f_1. As boundary conditions, we use the conditions of constancy and equality of the pressure at the ends of the lubricant layer:

$$\text{at} \quad x = x_0 \quad p = p_0; \quad \text{at} \quad x = x_L \quad p = p_0. \tag{40}$$

Use of the method of averaging [3] gives the same form of equation for pressure as (38), differing only in the numerical coefficients and the absence of a second term in the expression for F_1.

6. Example of Calculation. As an example of the use of the method of generalized similitude to calculate flows of gaseous lubricant, let us examine the problem of a lubricant wedge with a gap of linear form:

$$\delta = 1 - \alpha x, \quad \alpha = \frac{L}{\delta_0}\beta. \tag{41}$$

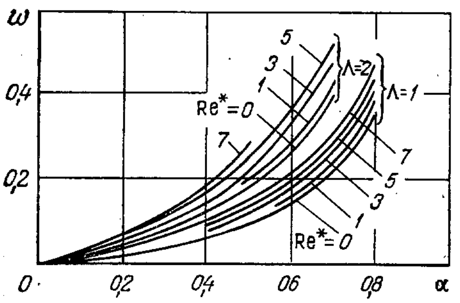

Fig. 2

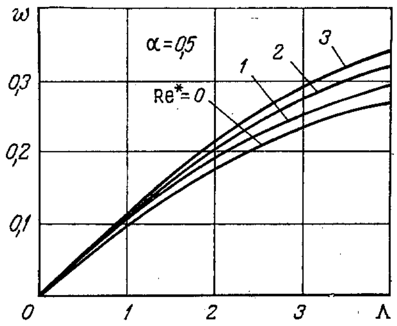

Fig. 3

Fig. 2. Dependence of buoyancy coefficient on wedge wetting angle.

Fig. 3. Dependence of buoyancy coefficient on compressibility factor.

For convenience, we will change over in Eq. (38) to the new function $\varphi = p\delta$ and write it in the form

$$A\varphi'' + B\varphi' + C\varphi = 0 \tag{42}$$

with the boundary conditions

$$\text{at } x = 0 \quad \varphi = 1; \quad \text{at } x = 1 \quad \varphi = 1 - \alpha. \tag{43}$$

We have the following expressions for the coefficients A, B, and C:

$$A = (1 - 2a_{21}\Lambda \mathrm{Re}^*)\delta\varphi F, \quad F = 1/F_1;$$
$$B = (1 - 2a_{21}\Lambda \mathrm{Re}^*)\delta\varphi' F + [(1 - 2a_{21}\Lambda \mathrm{Re}^*)(\delta F)' - 2\delta' F]\varphi - 2a_{10}\Lambda F;$$
$$C = -[2a_{10}\Lambda F' + \delta' F' \varphi]. \tag{44}$$

Equation (42) is nonlinear. We will solve it by the iteration method, calculating the coefficients A, B, and C in the preceding iteration. We begin the iteration with zero. The function φ is arbitrarily assigned as follows:

$$\varphi = 1 + x - (1 + \alpha)x^2. \tag{45}$$

For the numerical solution of Eq. (42), we replace it with the finite-difference analog

$$a_n\varphi_{n+1}^i + 2b_n\varphi_n^i + c_n\varphi_{n-1}^i = 0, \tag{46}$$

where

$$a_n = 2A_n + B_n\Delta\eta; \quad b_n = C\Delta\eta^2 - 2A; \quad c_n = 2A_n - B_n\Delta\eta. \tag{47}$$

The last equation was solved, on a grid with number of nodes N = 100 ($\Delta\eta = 0.01$), by the method of orthogonal trial runs until satisfaction of the condition

$$\max_{n=0}^{N} |\varphi_n^i - \varphi_n^{i-1}| \leqslant \varepsilon = 0{,}001, \tag{48}$$

where i is the number of the iteration.

The flow rate q in Eq. (39) for F_1, with satisfaction of the conditions of Rolle's theorem of differential calculus, is determined from Eqs. (33) at point x*, where dp/dx = 0.

The dimensionless value of buoyant force is determined from the known distribution of pressure along the lubricant wedge using the formula

$$w = \frac{W}{p_0 L} = \int_0^1 (\varphi/\delta - 1)\,dx. \tag{49}$$

Figures 1, 2, and 3 show the distribution of pressure and the coefficient of the vertical component of buoyancy for certain values of α, Λ, and Re*. These results show that the role of lubricant inertia leads to an increase in the ordinates of the pressure distribution, a certain shift in the pressure maximum in the direction of a smaller gap, and a consequent increase in buoyancy. Thus, e.g., the maximum increase in the buoyancy coefficient at $\alpha = 0.5$, $\Lambda = 1$, and $Re^* = 5$ is about 30% of its value at $Re^* = 0$.

NOTATION

u, v, components of lubricant velocity; x, y, Cartesian coordinates; p, pressure; ρ, density; μ, dynamic viscosity coefficient; h, enthalpy; δ, thickness of lubricant layer.

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JET FLOW OF SUPERCRITICAL AQUEOUS SOLUTIONS OF ELECTROLYTES WITH JOULE HEATING IN THIN CAPILLARY TUBES

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It was established that the heating of an aqueous solution of electrolyte in a thin capillary tube by a current passed through the solution is accompanied by movement of the liquid in the heated volume. Jet flow of the liquid was observed at certain values of the capillary-tube parameters and geometry.

The heating of an electrolyte solution in a capillary tube by a current passed through the solution leads to the onset of steady-state flow of the liquid.

The electrolytic cell used to study the properties of electrolyte solutions in the supercritical region takes the form of two glass vessels filled with the test liquid. The vessels are joined only by a thin ruby capillary tube. The electrodes are located a considerable distance from the tube. Watch-grade jewels of the STs type, made of synthetic ruby-10 (GOST 7137-73), were used for the capillary tubes. Ruby-10 can be soldered with molybdenum glass [1].

The capillary tube plays the role of concentrator of the electric field when a current is passed through the cell, so that the electrolyte is heated within the small ($\sim 10^{-5}$ cm^3) volume of the tube.

The pressure in the test liquid was greater than the critical value, amounting to $\sim$250 bar for aqueous solutions of LiCl, NaCl, and KCl at concentrations from 0.0125 to 0.05 M (molality) [2].

With the heating of the solution inside the capillary tube by an alternating current, one can observe transient ejections of heated liquid from the channel. At a certain value of supply voltage (with a certain amount of heating), these transients develop into steady-state flow in the form of a jet of heated liquid issuing from the capillary tube (Fig. 1).

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